Bloch waves homogenization and analysis of fluid-structure interactions

Mihaela Loredana Bălilescu

Abstract

The manuscript, prepared to defend my Habilitation thesis, contains a selection of my research results obtained in the fields of homogenization theory and analysis of fluid-structure interactions after defending my Ph.D. thesis. The main motivation is to very briefly describe the state of the art in these research fields, to give an overview of my contributions in these broad areas and to discuss some open problems and several perspectives I see for my future scientific and academic career.

The thesis is based on some of my original contributions to homogenization theory and fluid-structure interaction theory, contained in seventeen articles already published or submitted for publication in international journals with a broad audience, including not only mathematicians, but also physicists, engineers, among other scientists and one book. These results were obtained during the last twelve years of research in the following major areas: Bloch waves homogenization, where I concentrate on the effective or homogenized coefficients, as well as the Burnett or higher order macro coefficients, and analysis of fluid-structure interactions with focus on the existence and uniqueness of solutions, along with numerical aspects. Thus, the homogenization and fluid-structure interaction theories represent the core of my scientific work done during the last twelve years.

Apart from two short abstracts, one in Romanian and another one in English, the thesis contains two parts and a comprehensive bibliography.

The first part, structured into three chapters, is devoted to the presentation of my main scientific achievements since the completion of my Ph.D. thesis. After a brief introductory chapter presenting the state of the art in the fields of homogenization and fluid-structure interaction theories, the second chapter is divided into six distinct sections, summarizing my main contributions to homogenization theory. More precisely, some original results for the higher order macro coefficients or Burnett coefficients, as well as, for the effective or homogenized coefficients are presented. The chapter relies on the papers [2, 3, 4, 9, 11, 12, 15], proceedings [1, 4, 5] and the book [3] from the publications list.

Let me now describe each contribution corresponding to this second chapter. A first set of macro coefficients, known as the homogenized coefficients, appear in the homogenization of PDE on periodic structures. If energy is increased or scale is decreased, these coefficients do not provide adequate approximation. Using the Bloch decomposition, it is first realized that the above coefficients correspond to the lowest energy and the largest scale. This naturally paves the way to introduce other sets of macro coefficients corresponding to higher energies and lower scales which yield better approximation, known as Burnett coefficients or dispersion tensor. My task was to study and compare their properties with those of the homogenized coefficients.

In the first section, corresponding to paper [15], is studied the dependence of the effective and Burnett tensors in terms of the microstructure, considering the low contrast periodic media in one-dimensional and some laminated structures. Surprisingly, these tensors depend on the microstructure only through the local proportion parameter and in some cases, they do not depend on the microstructure at all. Going further with the pre-

vious analysis, in section two, corresponding to paper [12], it is described the set in which the Burnett tensor associated with a periodic structure lies, in one dimension, as the microstructure varies preserving the volume fraction. The proceedings [1, 4] review all these developments along with some new results. The object of discussion of the third section, corresponding to paper [9], is the variation on laminated microstructures of the higher order macro coefficients. Complete bounds are obtained on its quartic form along with the corresponding optimal structures. Using Blossoming Principle, it is shown that this coefficient is not negative in the sense of Legendre-Hadamard, even though its quartic form is negative. Generalizing the previous results, in section four, corresponding to paper [2], it was introduced the dispersion tensor or the Burnett coefficients in the class of generalized Hashin-Shtrikman microstructures. In the case of two-phase materials associated with the periodic Hashin-Shtrikman structures, the dispersion tensor has an unique minimizer, which is so called Apollonian-Hashin-Shtrikman microstructure.

Considering the effective or homogenized coefficients, in section five, corresponding to paper [11] and proceeding [5], using Bloch waves method for periodic microstructures, it was found the classical strange term in homogenization theory for PDE associated with the Laplace operator in $\mathbb{R}^N$ $(N = 3, 4, 5)$ in periodically perforated domain, and with homogeneous Dirichlet condition on the boundary of holes, as the size of holes goes to zero faster than the microstructure period. The difficulty of this problem, is the asymptotic expansion for all simple eigenvalues associated with Laplace operator in perforated domain, as the size of holes goes to zero. In the submitted paper [3] (partially discussed in section four of this second chapter), we went further and use spectral methods by introducing the Bloch waves to study the homogenization process in the non-periodic class of generalized Hashin-Shtrikman microstructures, which incorporates both translation and dilation with a family of scales. The classical homogenization result was established with providing the spectral representation of the homogenized coefficients. It offers a new lead towards extending the Bloch spectral analysis in the non-periodic, non-commutative class of microstructures. In section six, corresponding to the very recently submitted paper [2], the homogenization process of the fractional non-local elliptic boundary value problem, considering non-homogeneous Dirichlet type condition outside of a bounded domain $\mathcal{O} \subseteq \mathbb{R}^n$ was considered. By using the H-convergence method, under standard uniform ellipticity, boundedness and symmetry assumptions on coefficients, the effective coefficients were found. It was also proved that the the strange term does not appear in the homogenized problem associated with the fractional Laplace operator $(-\Delta)^s$ $(0 < s < 1)$ in a perforated domain. Both of these results have been obtained in the class of general microstructures.

The third chapter, divided into four distinct sections, is devoted to important contributions to fluid-structure interaction theory concerning numerical analysis of some relevant problems, as well as nice theoretical results on the existence and uniqueness of solutions. The chapter is based on the papers [1, 5, 6, 7, 8, 10] and proceeding [2] from publications list.

Let me now describe each contribution corresponding to this third chapter. In the first section, corresponding to paper [8], its short version [10] and proceeding paper [2], one propose a new characteristics method for the discretization of the two dimensional fluid-rigid body problem in the case where the densities of the fluid and the solid are different. The equations of the system are the Navier-Stokes equations in the fluid part, coupled with ordinary differential equations for the dynamics of the solid. The method is based on a global weak formulation involving only terms defined on the whole fluid-rigid domain. To take into account the material derivative, its was constructed a special characteristic function which maps the approximate rigid body at the discrete time level t_{k+1} into the approximate rigid body at time t_k. Convergence results are proved for both semi-discrete and fully-discrete schemes. Further, in the second section, corresponding to paper [7], was considered a Lagrange-Galerkin scheme to approximate a two dimensional fluid-deformable structure interaction problem. Numerical schemes based on the use of the characteristics method for a deformable solid were introduced and convergence results were stated for both semi and fully discrete schemes. In the working paper [1], will soon appear the complete proofs of these results.

Going further, a new model for the motion of a viscous incompressible fluid was proposed in paper [6] and is partially written in section two of this chapter. More precisely, was considered the Navier–Stokes system with a boundary condition governed by the Coulomb friction law. With this boundary condition, the fluid can slip on the boundary if the tangential component of the stress tensor is too large. In the opposite case, we recover the standard Dirichlet boundary condition. The existence and uniqueness of weak solution in the two–dimensional problem and the existence of at least one solution in the three–dimensional case, together with regularity properties and an energy estimate were proved. A fully discrete scheme of the problem using the characteristic method and numerical simulations in two physical examples were done. Generalizing the previous result, in section four, corresponding to paper [5], a new model in a fluid-rigid structure system composed by a rigid body and a viscous incompressible fluid using a boundary condition based on Coulomb's law was proposed. The governing equations were considered the Navier-Stokes system for the fluid and the Newton laws for the body. The corresponding coupled system can be written as a variational inequality. It was proved that there exists a weak solution of this system.

The second part of this thesis presents some career evolution and development plans. After a brief review of my scientific and academic background, further research directions and some future plans on my scientific and academic career are presented. I will discuss some short, medium and long term development plans. A brief description of some open questions I would like to study in the future will be made, as well.

The thesis ends by a comprehensive bibliography, illustrating the state of the art in these vast fields of homogenization and fluid-structure interaction theories.

My major original contributions contained in this habilitation thesis can be summarized as follows:

- performing a rigorous study of the Burnett coefficients in periodic structures and in the class of generalized Hashin-Shtrikman microstructures;
- getting original results on the unique minimizer called Apollonian-Hashin-Shtrikman microstructure for the dispersion tensor, in the case of two-phase materials associated with the periodic Hashin-Shtrikman structures;
- obtaining homogenization results by using the spectral method of Bloch waves for problems in perforated media;
- obtaining new homogenization results for the fractional non-local elliptic boundary value problem, considering non-homogeneous Dirichlet type condition outside of a bounded domain;
- elaborating a new characteristics method for the discretization of the two dimensional fluid-rigid body problem in the case where the densities of the fluid and the solid are different and prove its convergence;

- constructing convergent numerical schemes based on the use of the characteristics method for deformable solids and prove their convergence;
- deriving a new model for the motion of a viscous incompressible fluid, considering the Navier–Stokes system with the Coulomb friction law on the boundary;
- obtaining the existence and uniqueness (in some cases) for the new mathematical model for the motion of a viscous incompressible fluid using the Coulomb friction law on the boundary;
- performing the proof of the existence of a weak solution for a new model in a fluid-rigid structure system composed by a rigid body and a viscous incompressible fluid using the Coulomb's law.

The results included in this thesis have been obtained in close collaboration with several academic and research institutions from Romania and abroad: University of Pitești, University of Chile, University of Lorraine, Tata Institute of Fundamental Research in Bangalore, Federal University of Santa Catarina, University of Washington, University of Pau and Pays de l'Adour. I am grateful to all my co-authors, Professor C. Conca, A. Ghosh, Dr. T. Ghosh, Professor J. San Martín, Professor J.-F. Scheid, Professor T. Takahashi, and Professor M. Vanninathan for a nice collaboration, for their important contribution to our papers, and for useful advices and interesting discussions. I hope that all these results might open new and promising perspectives for further developments and future collaborations with well-known scientists from Romania and from abroad.